High Dimensional Predictive Inference

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Abstract

Let $X \mid \mu \sim N_p(\mu, v_x I)$ and $Y \mid \mu \sim N_p(\mu, v_y I)$ be independent p-dimensional multivariate normal vectors with common unknown mean μ. Based on only observing $X = x$, we consider the problem of obtaining a predictive density $\hat{p}(y \mid x)$ for Y that is close to $p(y \mid \mu)$ as measured by expected Kullback-Leibler loss. This is the predictive version of the general problem of estimating μ under quadratic loss, and we see that a strikingly parallel theory exists for addressing it.

To begin with, a natural "straw man" procedure for this problem is the (formal) Bayes predictive density $\hat{p}_U(y \,|\, x)$ under the uniform prior $\pi_U(\mu) \equiv 1$, which is best invariant and minimax. It turns out that there are wide classes procedures that dominate $\hat{p}_U(y \mid x)$ including Bayes predictive densities under superharmonic priors. Indeed, any Bayes predictive density will be minimax if it is obtained by a prior yielding a marginal that is superharmonic or whose square root is superharmonic. For the characterization of admissible procedures for this problem, the class of all generalized Bayes rules here is seen to form a complete class, and easily interpretable conditions are seen to be sufficient for the admissibility of a formal Bayes rule.

Moving on to the multiple regression setting where we observe $X \sim N_m(A\beta, \sigma^2 I)$ and would like to estimate the predictive density $p(y \mid \beta)$ of a future $Y \sim N_n(B\beta, \sigma^2 I)$, our results are seen to extend naturally. Going further, we address the situation where there is model uncertainty and only an unknown subset of the predictors in A is thought to be potentially irrelevant. For this purpose, we develop multiple shrinkage predictive estimators along with general minimaxity conditions. Finally, we provide an explicit example of a minimax multiple shrinkage predictive estimator based on scaled harmonic priors. (This is joint work with Larry Brown, Feng Liang and Xinyi Xu).