Variance Function Estimation via Model Selection

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We consider a problem of estimating an unknown variance function $\sigma^2(\cdot)$ in a random heteroscedastic regression model $Y = m(X) + \sigma(X)\varepsilon$. Both the regression function and the logarithm of the variance function is modelled by piecewise polynomials. A finite collection of such parametric models pertaining to a family of partitions of support of explanatory random variable X is studied. Penalized model selection criteria as well as post-model-selection estimates are introduced based on Maximum Likelihood (ML) or Restricted Maximum Likelihood (REML) method of estimation of the involved parameters. The estimators are defined as ML or REML estimators in the models with dimensions chosen by respective selection rules. We discuss consistency properties of the solution pertaining to ML estimation when distribution of (X, Y) belongs to one of the considered models as well as its behaviour under misspecification. Some encouraging results of simulations in which proposed estimator is compared to a two-stage local linear estimator of the variance function will be presented.