Adaptive Estimation of the Distribution Function and its Density in Sup-Norm Loss (joint work with E. Giné)

Abstract: Given an i.i.d. sample from a distribution F on $\mathbb{R}$ with uniformly continuous density p_0, purely-data driven estimators are constructed that efficiently estimate F in sup-norm loss, and simultaneously estimate p_0 at the best possible rate of convergence over Hölder balls, also in sup-norm loss. The estimators are obtained from applying a model selection procedure close to Lepski's method with random thresholds to projections of the empirical measure onto spaces spanned by wavelets or B-splines. We obtain explicit constants in the asymptotic risk of the estimator, as well as oracle-type inequalities in sup-norm loss. The random thresholds are based on suprema of Rademacher processes indexed by wavelet or spline projection kernels. We also derive Bernstein-analogues of the inequalities in Koltchinskii (2006) for the deviation of suprema of empirical processes from their Rademacher symmetrizations.