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"On the Distribution of the Adaptive LASSO Estimator"

We study the distribution of the adaptive LASSO estimator (Zou, 2006) in finite-samples as well as in the large-sample limit. The large-sample distributions are derived both for the case where the adaptive LASSO estimator is tuned to perform conservative model selection as well as for the case where the tuning results in consistent model selection. We show that the finite-sample as well as the large-sample distributions are typically highly non-normal, regardless of the choice of the tuning parameter. The uniform convergence rate is also obtained, and is shown to be slower than $n^{-1/2}$ in case the estimator is tuned to perform consistent model selection. In particular, these results question the statistical relevance of the `oracle' property of the adaptive LASSO estimator established in Zou (2006). We also present simulation results that confirm our theoretical analysis in an extended model. Finally, we provide an impossibility result regarding the estimation of the distribution function of the adaptive LASSO estimator.