Sparse Exponential Weighting
as an alternative to LASSO and Dantzig selector

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The quality of solving several statistical problems, such as adaptive nonparametric estimation, aggregation of estimators and estimation under the sparsity scenario can be assessed in terms of sparsity oracle inequalities (SOI) for the prediction risk. One of the challenges is to build estimators that attain the sharpest SOI under minimal assumptions on the dictionary. Methods of estimation adapted to the sparsity scenario are mainly of the two types. Some of them, like the BIC, enjoy nice theoretical properties without any assumption on the dictionary but are computationally infeasible starting from relatively modest dimensions p. Others, like the Lasso or the Dantzig selector, can be easily realized for very large p but their theoretical performance is conditioned by severe restrictions on the dictionary. We will focus on Sparse Exponential Weighting, a new method of sparse recovery realizing a compromise between theoretical properties and computational efficiency. The theoretical performance of the method in terms of SOI is comparable with that of the BIC and even better in some aspects. No assumption on the dictionary is required. At the same time, the method is computationally feasible for relatively large dimensions p. It is constructed using an exponential weighting with suitably chosen priors, and its analysis is based on the PAC-Bayesian ideas in statistical learning. In particular, we obtain some new PAC-Bayesian bounds with leading constant 1 and we develop a general technique to derive sparsity oracle inequalities from the PAC-Bayesian bounds. This is a joint work with Arnak Dalalyan and Philippe Rigollet.